Performance Limits of an Aircraft Inertial-Based Stochastic Lateral Guidance System

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The performance limits that can be achieved for a specified level of control activity are explored for a class of incomplete state feedback inertial data based lateral control systems in a stochastic gust environment. The stochastic optimization problem is defined and direct minimization of the performance index, consisting of weighted elements of the system covariance matrix, is carried out using parameter optimization techniques. Optimal solutions are presented for an inertial measurement based reduced state feedback lateral control system for the Convair 880 transport aircraft. The solutions provide a lower bound on the rms path deviation due to wind gusts which can be achieved with a specified level of control activity and also provide valuable guidance in the choice of primary lateral effector between ailerons and differential spoiler. The performance characteristics of a conventional radio coupler design for the CV880 are presented for comparison purposes.

Nomenclature

p = vector of adjustable parameters
x = n-dimensional state
u = m-dimensional Gaussian white noise
α = n-dimensional weighting vector
C = weighting matrix
F = matrix associated with the linear dynamical system; F is a function of the parameter vector p
G = matrix specifying the coupling between the system and the stochastic disturbance vector u
P = matrix of Lagrange multipliers
Q = covariance matrix of the white noise process
X = covariance matrix of the system state vector x
g = acceleration due to gravity
J = performance index
p = roll rate
r = yaw rate
v_p = aircraft path velocity
w_y = weight on covariance of lateral position error
w_δ = weight on covariance of effector activity
y = lateral path error
β = sideslip angle
δ = effector deflection
ϕ = roll attitude
ψ = heading angle

Subscripts

a = aileron
c = command
i = ith element of vector
ij = i-row j-column element of matrix
n = aerodynamic noise
r = rudder
s = differential spoiler

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Introduction

INERTIAL navigators are becoming a common instrument on transport aircraft. In addition to its navigation value, inertial position, velocity and acceleration information, suitably bounded in error by radio measurement, is ideal for application in flight-path control systems.

The achievement of accurate lateral guidance of an aircraft in an environment subject to stochastic disturbances (wind gusts) and limitations on expended control effort is an important step to the development of future high-precision air traffic control. While the best solutions are provided within the framework of classical optimal control theory, the solution of problems associated with the measurement of the complete vehicle state, including correlated noise, or the estimation of the missing state elements using, for example, a Luenberger "observer"[1] can result in a complicated design. As a result a reduced state feedback control configuration is often sought if satisfactory performance can be achieved.

Optimal reduced state feedback stochastic controllers are more difficult to synthesize than complete feedback controllers, in general. Solution for the optimal gains may be found by direct minimization of the performance index using parameter optimization techniques[5-7] or by solving a two-point boundary value problem.[2] The former procedure has been utilized here. The resultant parameter optimized solutions provide a useful bound on control system performance.

Stochastic Control System Design by Parameter Optimization

An outstanding problem associated with the design of aerodynamic vehicle control systems is the large number of parameters which commonly define the control law. This complexity is a result of the number of available feedback variables and a variety of effectors. Such multiplicity results in an extremely tedious design process if conventional cut-and-try procedures are applied. To circumvent this difficulty, systematic parameter optimization techniques are utilized. The solutions generated by parameter optimization are optimal with respect to the selected performance index. By suitably scanning the performance index basic performance limitations associated with the selected control law structure, effector size and type, and control energy limits may be identified.

In order to facilitate this analysis, the characteristics of the response of a linear system to stochastic inputs must be delineated mathematically. Consider the system of linear differential equations:

$$\dot{x} = Fx + Gu \tag{1}$$

u is assumed to be uncorrelated with the state x.

The covariance matrix X of x is defined by the relationship

$$X = E(xx') \tag{2}$$

where E is the mathematical expectation operator. It is apparent that X is a symmetic matrix, a property which may be used advantageously in computations.

Of prime interest in control system investigations are time-invariant or stationary systems. (Such an assumption is valid over a small range of vehicle velocities. Each speed regime must be investigated separately and control system parameter values suitably scheduled.) A linear system of the form Eq. (1) is time-invariant if the matrices F and G are constant. If the system is time-invariant and asymptotically stable, and if the correlation matrix Q is also constant, the matrix X will approach a constant as $t \to \infty$. This implies that the derivative $\dot{X}$ of X vanishes as $t \to \infty$ or that the final value of X satisfies the set of linear algebraic equations

$$FX + XF' + GQG' = 0 \tag{3}$$

The process is then said to be statistically stationary in the limit $t \to \infty$.

The solution of Eq. (3) is conveniently obtained by transformation to a set of $n(n+1)/2$ ordinary linear algebraic equations which are then solved using any one of a multitude of standard techniques.

Since the diagonal terms of X represent the mean-square values of the state elements responding to the stochastic disturbance, X provides the basis for formulating an optimization problem which leads to the minimization of system response to stochastic inputs subject to penalties on the expended control effort.

Let the performance index, J, be defined as a linear combination of the diagonal elements of the covariance matrix, X. Such a performance index may be expressed in the form[5]

$$J = \sum_{i=1}^{n} \alpha_i x_{ii} \tag{4}$$

where

$$\alpha_i \geq 0 \qquad i = 1,n \tag{5}$$

The elements of α are selected to reflect the control goal. For example, the association of nonzero values of α_i with the trajectory error and the effector output results in a solution which minimizes the mean-square value of the trajectory error subject to a penalty on effector activity.

For analytical purposes Eq. (4) is conveniently expressed in the equivalent form

$$J = \text{trace } CX \tag{6}$$

where

$$\text{trace } CX = \sum_{i=1}^{n} (CX)_{ii} \tag{7}$$

and

$$c_{ij} = \begin{cases} 0 & i \neq j \\ \alpha_i & i = j \end{cases} \tag{8}$$

If it is assumed that the system of Eq. (1) is stationary so that X is the solution of Eq. (3) the optimization problem may be described.

Definition of the Problem

Find a set of parameters, p, which minimizes the performance index

$$J = \text{trace } CX \tag{9}$$

subject to the constraint

$$FX + XF' + GQG' = 0 \tag{10}$$

Equation (10) is conveniently handled by adjoining the constraints to the performance index through the artifice of a Lagrange multiplier matrix P. The performance index is then written

$$J = \text{trace } [CX + P(FX + XF' + GQG')] \tag{11}$$

The first variation in the performance index may then be written by considering perturbations in P, X, and p.

In order for the first variation to vanish with respect to arbitrary perturbations in δp, δX, and δP, the following set of Canonical equations of the first variation must be satisfied.

Canonical equations of the first variation

$$FX + XF' + GQG' = 0 \tag{12}$$

$$PF + F'P + C = 0 \tag{13}$$

$$J_p = 0 \tag{14}$$

where J_p is the gradient of J with respect to p. The ith element of J_p is

$$J_{p_i} = \text{trace } 2\, PX(\partial/\partial p_i)F' \tag{15}$$

Since the matrices $(\partial/\partial p_i)F'$ are relatively easy to compute, Eq. (15) provides a convenient basis for evaluating the gradient, J_p, of the performance index.

The generation of weak relative minima is accomplished by a series of systematic operations which lead to a solution of the Canonical Equations (12–14). Simultaneous solution of Eq. (12) to Eq. (14) is generally not attempted; however, Eq. (12) to Eq. (13) are satisfied in each iteration. A description of the more common parameter optimization algorithms is found in Ref. 7 with an illustrative example. The solutions presented in this paper were obtained using an accelerated version of the method of Steepest Descent.

Application to a Class of Inertial Lateral Control Systems for the Convair 880

Control of the position of the aircraft in the horizontal plane relative to the desired path is accomplished by performing coordinated turns. If an aerodynamic vehicle is rolled about its longitudinal axis, a horizontal component of the lift vector results. If the sideslip angle is maintained at zero, a yaw rate must be established to maintain equilibrium. The resultant yaw rate alters the direction of the velocity vector. If ψ_m is the heading reference, the lateral velocity of the aircraft relative to the path centerline is

$$\dot{y} \approx v_p \sin(\psi - \psi_m) \tag{16}$$

Roll angle control is achieved by establishing moments about the longitudinal axis. Such moments may be produced by ailerons or spoilers, operated differentially.

Ailerons and spoilers are equally effective for controlling roll rate. Turn coordination, however, is more simply produced with differential spoilers which results in a drag-produced yawing moment which aids the establishment of the desired yaw rate.

Since the effectiveness of ailerons varies as the square of the airspeed, it is often essential to provide spoiler augmentation to achieve adequate levels of low-speed lateral control.

The CV880 utilizes ailerons as well as differential spoilers for low-speed lateral control, thus a hybrid lateral control system will ultimately be used for this vehicle.

Ailerons and spoilers are quite similar from the roll-dynamics point of view. Thus, it was decided at the outset to use similar control laws for both effectors. The aileron and spoiler effector commands are linear combinations of lateral position, velocity and acceleration and roll angle and roll rate errors:

$$\begin{aligned} \delta_{ac} &= p_1 y + p_2 \dot{y} + p_3 \ddot{y} + p_4 \phi + p_5 p \\ \delta_{sc} &= p_6 y + p_7 \dot{y} + p_8 \ddot{y} + p_9 \phi + p_{10} p \end{aligned} \tag{17}$$

Thus, up to 10 parameters must be defined during control synthesis. In addition, turn coordination must be assured by the computation of appropriate rudder commands.

Turn coordination was provided by closure of an additional control loop on yaw rate which also provides dutch-roll mode damping. The desired roll angle, ϕ_c, is

$$\phi_c = [p_1 y + p_2 \dot{y} + p_3 \ddot{y}]\, 1/p_4$$

or

$$\phi_c = [p_6 y + p_7 \dot{y} + p_8 \ddot{y}]\, 1/p_9 \tag{18}$$

so that the coordinated turn rate is

$$r_c \approx g\phi_c / v_p \tag{19}$$

If the rudder command is

$$\delta_{rc} = -p_{11}[r_c - r] \tag{20}$$

the rudder will operate to make $r \approx r_c$.

The translation-error variables y, $\dot{y}$ and $\ddot{y}$ are provided by the integrated IMU-Radio Aid navigation system discussed in Ref. 6. Roll angle is provided by processing IMU gimbal angles (or from the vertical gyroscope) and roll rate is usually measured with a body-mounted rate gyro.

The effector commands are fed to the control surface actuators which are modeled by first-order lags. The surface deflections are inputs to a set of linear vehicle equations which are detailed in Ref. 6.

The optimization problem was formulated for a lateral control system using ailerons or differential spoilers. The performance index reflects concern with the path deviation y and the spoiler δ_s or aileron δ_a deflections

$$J = w_y E(y^2) + w_\delta E(\delta^2) \tag{21}$$

where δ represents δ_s or δ_a. The role of δ is determined by constraints placed on the elements of the p vector. The value of w_y was held constant while w_δ was varied to explore a range of solutions.

The lateral vehicle dynamics, control laws, first-order effector models, and exponentially correlated noise were written in the form of Eq. (1) for the final approach flight configuration. The state vector and matrices associated with the problem are shown in Fig. 1.

The stochastic environment was characterized by a 10 fps rms gust velocity, corresponding to a strong turbulent condition. The resultant rms value of the aerodynamic noise component β_n of β is approximately 2.0 degrees rms for an approach airspeed of 280 fps. The covariance matrix Q required to achieve the desired rms sideslip angle is

$$Q = [10.0] \tag{22}$$

$$F = \begin{bmatrix} -1.52 & 0.473 & -2.89 & 0 & 0 & 5.18 & F_{17} & -0.483 & 0 \\ -0.0626 & -0.1653 & 0.653 & 0 & 0 & -1.17 & F_{27} & -0.3746 & 0 \\ 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 \\ F_{61} & F_{62} & F_{63} & F_{64} & F_{65} & F_{66} & F_{67} & -1.7886 & 0.581 \\ F_{71} & 0 & F_{73} & F_{74} & F_{75} & F_{76} & -10.0 & 0 & 0 \\ 0 & 10.0p_{11} & 0 & F_{84} & F_{85} & F_{86} & 0 & -10.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.807 \end{bmatrix}$$

Spoilers Only Matrix		Ailerons Only Matrix	
$F_{17} = 0.9052$	$F_{71} = 10.0p_{10}$	$F_{17} = -0.73$	$F_{71} = 10.0p_5$
$F_{27} = 0.0482$	$F_{73} = 10.0p_9$	$F_{27} = -0.1045$	$F_{73} = 10.0p_4$
$F_{61} = 0.5541 - 0.259p_{10}$	$F_{74} = 10.0p_6$	$F_{61} = 0.5541$	$F_{74} = 10.0p_1$
$F_{62} = 0.7089 + 1.765p_{11}$	$F_{75} = 10.0p_7$	$F_{62} = 0.7089 + 1.765p_{11}$	$F_{75} = 10.0p_2$
$F_{63} = 0.0412 - 0.259p_9$	$F_{76} = 10.0p_8$	$F_{63} = 0.0412$	$F_{76} = 10.0p_3$
$F_{64} = -0.203p_{11}p_6/p_9 - 0.259p_6$	$F_{84} = -1.15p_6p_{11}/p_9$	$F_{64} = -0.203p_{11}p_1/p_4$	$F_{84} = -1.15p_1p_{11}/p_4$
$F_{65} = -0.203p_{11}p_7/p_9 - 0.259p_7$	$F_{85} = -1.15p_7p_{11}/p_9$	$F_{65} = -0.203p_{11}p_2/p_4$	$F_{85} = -1.15p_2p_{11}/p_4$
$F_{66} = -0.2227 - 0.203p_{11}p_8/p_9 - 0.259p_8$	$F_{86} = -1.15p_8p_{11}/p_9$	$F_{66} = -0.2227 - 0.203p_{11}p_3/p_4$	$F_{86} = -1.15p_3p_{11}/p_4$

$$x = \begin{bmatrix} p \\ r \\ \varphi \\ y \\ \dot{y} \\ \ddot{y} \\ \delta \\ \delta_r \\ \beta_n \end{bmatrix} \qquad G = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.581 \\ 0 \\ 0 \\ 0.807 \end{bmatrix} \qquad u = [w]$$

Fig. 1 The state vector and the *F*, *G* and *u* matrices of the Linear Dynamical System.

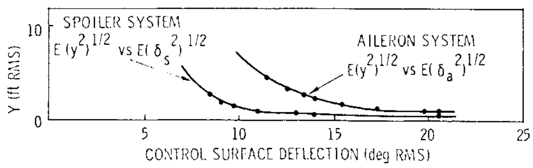

Fig. 2 ***RMS*** **lateral error vs rms effector deflection for the aileron and spoiler lateral position control systems.**

Two different lateral control system configurations were investigated: 1) aileron and rudder only ($p_6 \rightarrow p_{10} = 0$); and 2) differential spoiler and rudder only, ($p_1 \rightarrow p_5 = 0$). The first design utilizes the most common method—ailerons to control the roll attitude of the vehicle. The second design exploits the moment produced by differential operation of the spoilers for roll control.

Operation of the ailerons produces an adverse yawing moment as a result of the larger drag produced by the downward-deflected aileron. This leads to a yaw rate which tends to uncoordinate the turn ($|\beta| > 0$). The yawing moment is normally counteracted by a rudder deflection which produces a counter moment. The resultant side force on the vertical stabilizer, under these circumstances, tends to increase the lateral error that the control system is attempting to correct. As a result it was anticipated that the performance of the aileron control system would be somewhat inferior to that of a spoiler-based lateral position control system. This conclusion was validated in practice, as shown in Fig. 2. The spoiler control system reduces the rms lateral error by a factor of 2 compared to the aileron system.

It is of interest to compare the Lear-Siegler Autoland lateral control design[8] with the optimized lateral systems. The Lear configuration uses spoiler and aileron simultaneously. As a result it is not possible to achieve a direct comparison. However, by omitting the acceleration feedback gain in the optimized configurations and appropriately adjusting the other parameter values it is possible to achieve aileron-only and spoiler-only designs which closely approximate the performance of the Lear design. The required parameter values are listed in Tables 1 and 2. The reference aileron system achieves a lateral position error of 39 ft rms for effector activity of 5° rms. The corresponding reference spoiler system achieves a 53 ft rms position error for an effector activity of 3° rms. These results may be compared to those, shown in Fig. 2, achieved with the optimized systems. As a result of sensor noise and dynamic limitations it is only possible to make marginal improvements in the Lear localizer coupler design. Thus it is apparent that the optimized inertial control systems can potentially reduce the rms lateral error due to aerodynamic disturbances by a factor of 10–15 times, depending on the permissible level of control activity and the effector choice.

The parameter values associated with the optimized systems are illustrated in Figs. 3–6. Both optimal configurations feature roll angle and rate gains which are significantly greater than those associated with the reference systems. The position, velocity, and acceleration gain characteristics are roughly the same for both systems and much larger than the reference values. The difference between the aileron and spoiler effectors, mentioned previously, is clearly reflected in the yaw damper gain characteristics; significantly lower values are used in the spoiler system, particularly at high performance levels.

Stochastic and transient responses of the optimized lateral aileron only control system are seen in Figs. 7 and 8, respectively, produced by the CV880 digital simulation. The stochastic results are in response to lateral gusts (rms value

Table 1 Aileron reference model gains

Parameter	Description	Value
p_1	lateral position gain	0.0574
p_2	lateral velocity gain	0.400
p_3	lateral acceleration gain	0.000
p_4	roll angle gain	2.560
p_5	roll rate gain	1.580
p_{11}	yaw damper gain	3.940

Table 2 Spoiler reference model gains

Parameter	Description	Value
p_6	lateral position gain	−0.019
p_7	lateral velocity gain	0.100
p_8	lateral acceleration gain	0.000
p_9	roll angle gain	−0.853
p_{10}	roll rate gain	−0.523
p_{11}	yaw damper gain	3.940

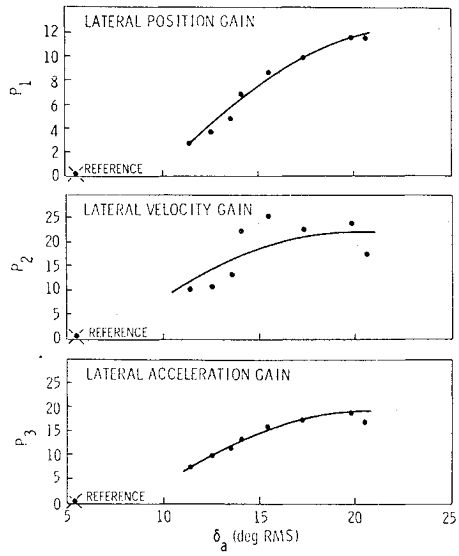

Fig. 3 Lateral position, velocity and acceleration gains vs rms aileron deflection for the aileron lateral position control system—the reference system gains are indicated with crosses.

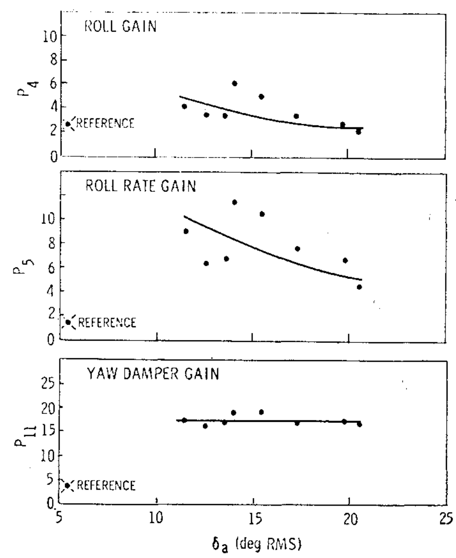

Fig. 4 Roll angle, rate and yaw damper gain vs rms aileron deflection —the reference system gains are indicated by crosses.

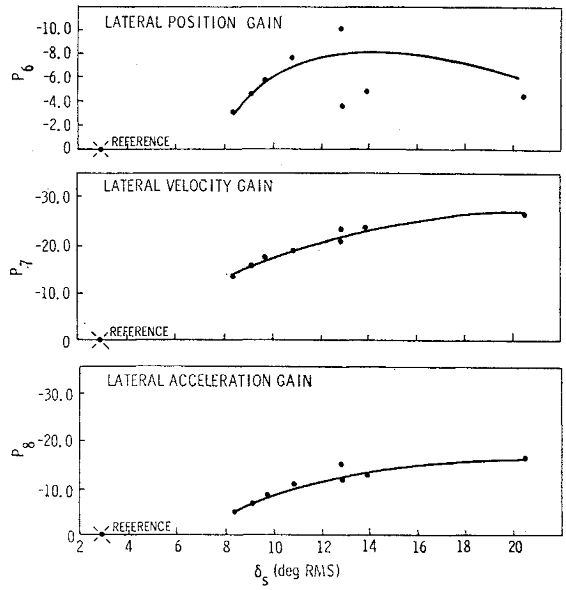

Fig. 5 Lateral position, velocity and acceleration gains vs rms spoiler deflection for the spoiler lateral control system—the reference system gain values are indicated by crosses.

equal to 10 fps) represented by equivalent sideslip noise, β_n. The responses of three systems, each optimized for a different value of effector weighting coefficient, are displayed. Decreased rms trajectory error is associated with increased effector activity. The transient position error response is fast and well-damped with characteristic high-frequency components for the high-gain systems apparent in the remaining aircraft state.

The spoiler only control system responses shown in Figs. 9 and 10 are similar to those of the aileron system, but, in general, display better performance. RMS trajectory errors are less than for the aileron system and the associated rms vehicle state errors are also less. Transient response characteristics for the spoiler system are comparable to those for the aileron system.

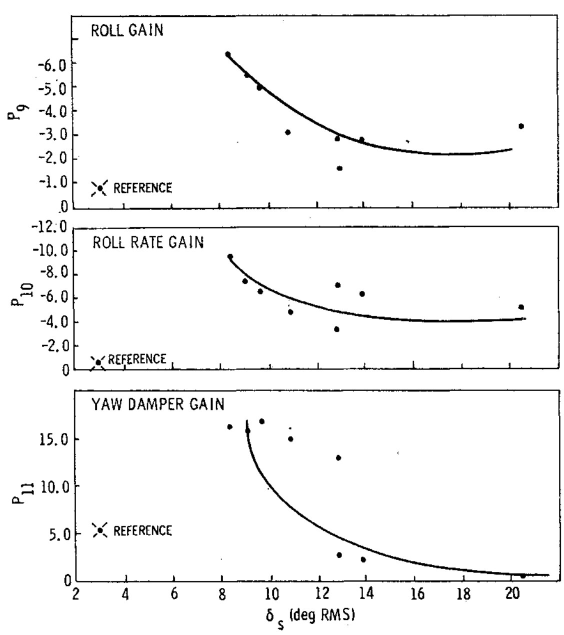

Fig. 6 Roll angle, rate and yaw damper gains for the spoiler lateral position control system—the reference system gains are indicated by crosses.

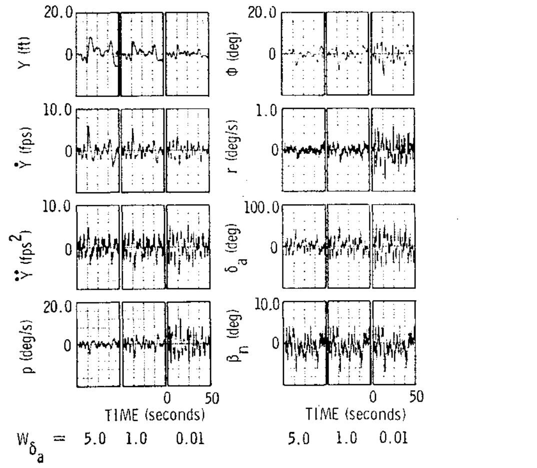

Fig. 7 Response of the optimized aileron only system to lateral gusts (rms value = 10 fps).

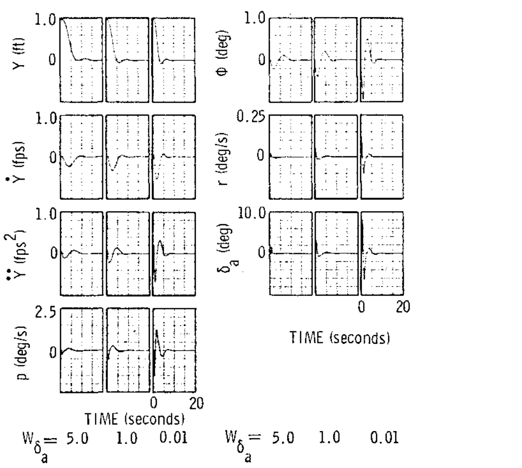

Fig. 8 Response of the optimized aileron only system to an initial error in Y for three values of the aileron weighting coefficient—the lateral position error weighting coefficient was constant at $w_y = 10.0$.

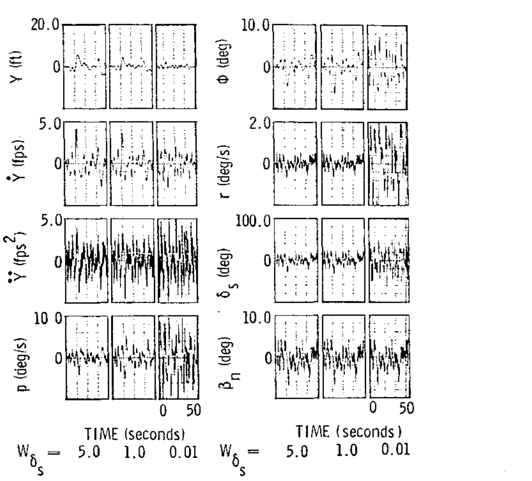

Fig. 9 Response of the optimized lateral spoiler only control system to lateral gusts (rms value = 10 fps).

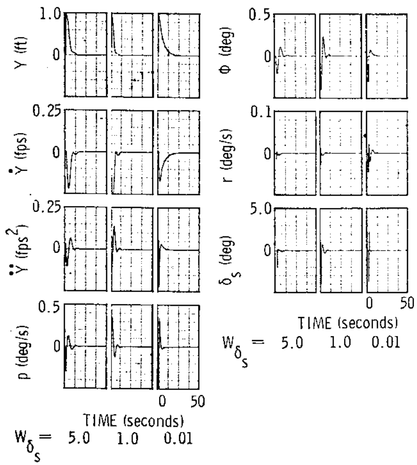

Fig. 10 Response of the optimized lateral spoiler only control system to an initial error in lateral position for three values of the spoiler weighting coefficient—the lateral position weighting coefficient was constant at $w_y = 10.0$.

Conclusions

The exploitation of inertial system data can result in significant reductions in lateral path control system sensitivity to environmental disturbances and improved transient response characteristics.

While the optimized inertially based control solutions provide a lower bound on the rms lateral trajectory error that can be achieved with a specified level of control activity, it is apparent that the error realized in practice will be higher for a number of reasons. 1) Radio measurement noise introduced during inertial system error correction will produce path deviations. 2) It may not be possible to realize the full increases in control law parameter values called for by the optimized solutions due to limitations imposed by structural flexibility not considered here. 3) Lower gains may be required to assure satisfactory operation for all possible variations in vehicle parameters in the landing approach regime. 4) The discrete character of the inertial measurement noise and time lags.

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